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Generalized Newtonian Viscosity Functions for Hydrodynamic Lubrication

An improved viscosity function multiplying two modified Carreau forms is offered which describes the shear-dependence of the polymer and of the base oil without the artifacts associated with a sum of individual viscosity functions.

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Session: 3C Fluid Film Bearings I, 101C, Session Chair: Michel Fillon
Tuesday, May 22, 2018, 9:30 AM to 10:00 AM

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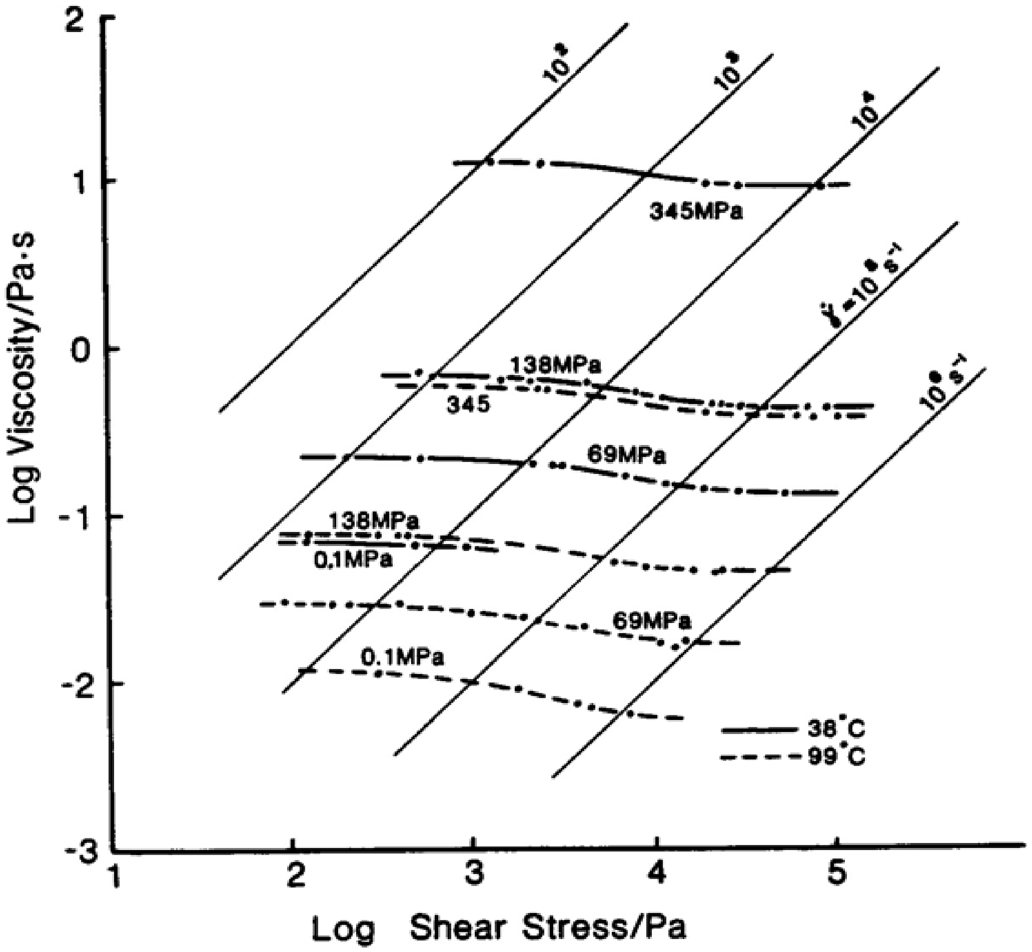
The author has 44 years of experience in measuring shear-dependent viscosity in liquid lubricants at both ambient and high pressures.

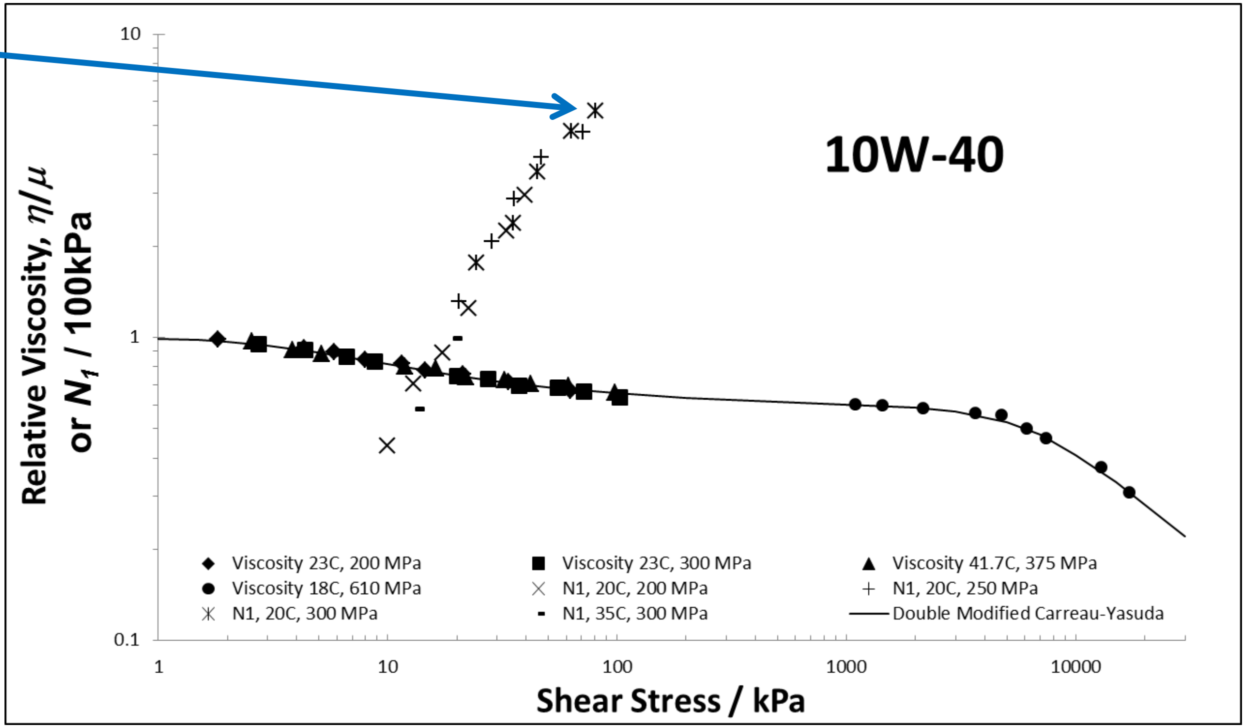
I have seen just about everything.

Flow curves generated at GA Tech 50 years ago!



When only the shear-dependence of the viscosity is addressed, we are talking about **Generalized Newtonian** behavior.

The Primary Normal Stress Difference is rising very fast when the viscosity begins to decrease with shear stress and here it reaches to almost 1 MPa.



Shear-thinning may not be the most interesting aspect of non-Newtonian response of a VI improved oil.

The compact master curve shown for the viscosity results from time-temperature-pressure superposition.

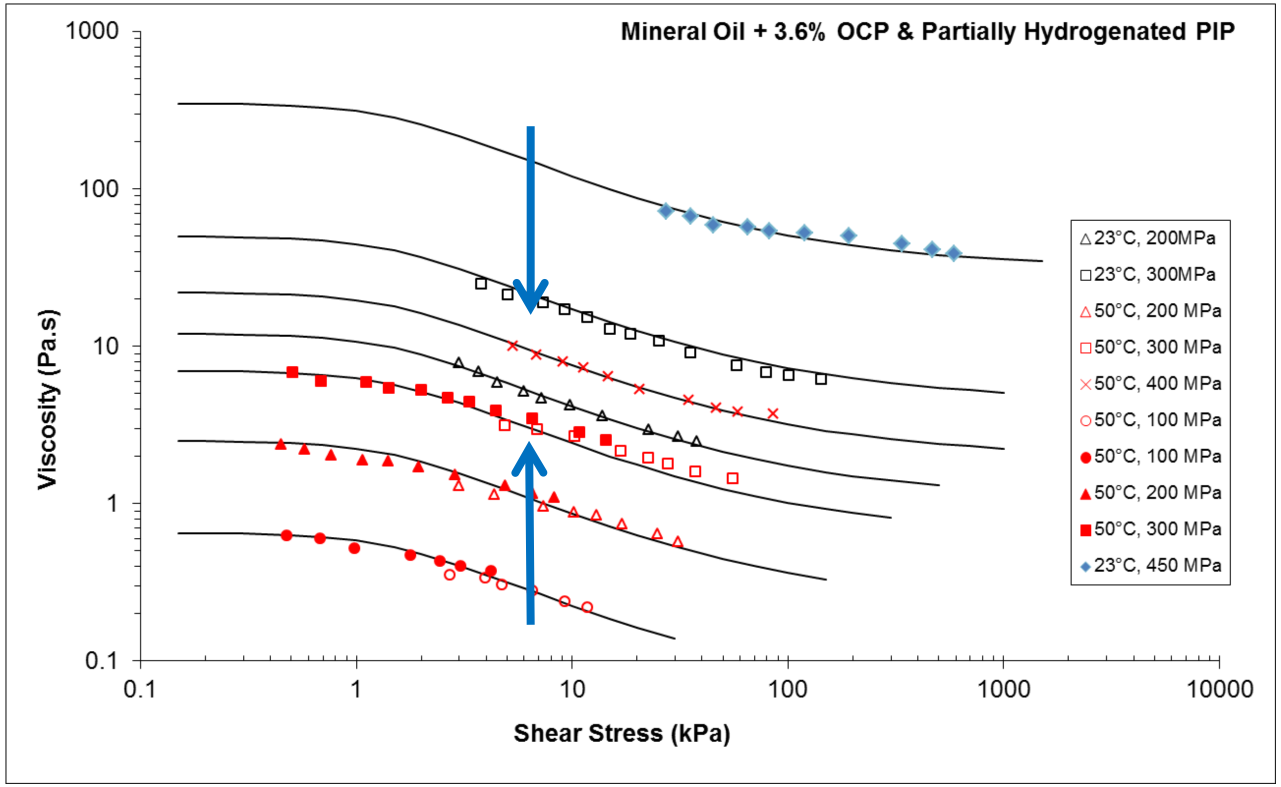
The Primary Normal Stress Difference is the extra tensile stress in the flow direction. The Navier-Stokes (and ordinary Reynolds) equations are valid for Generalized Newtonian but not for non-Newtonian flows in general.

“The complete definition of a Newtonian
fluid requires it to satisfy the complete Navier-Stokes equations rather than simply
exhibiting a constant value of shear viscosity.”

Chhabra, R. P. (2010). Non-Newtonian fluids: an introduction. In *Rheology of complex fluids* (pp. 3-34). Springer, New York, NY.

Time-temperature-pressure superposition is the means by which viscometer measurements may be applied to real tribology problems.

The viscosity function, $\eta(\dot{\gamma})$ or $\eta(\tau)$, need not be measured at all temperatures and pressures.



Flow curves may be shifted vertically to form a master curve.

This is why plotting viscosity versus stress is preferred when temperature and pressure vary.

There are barriers to the characterization of shear-thinning at ambient pressures.

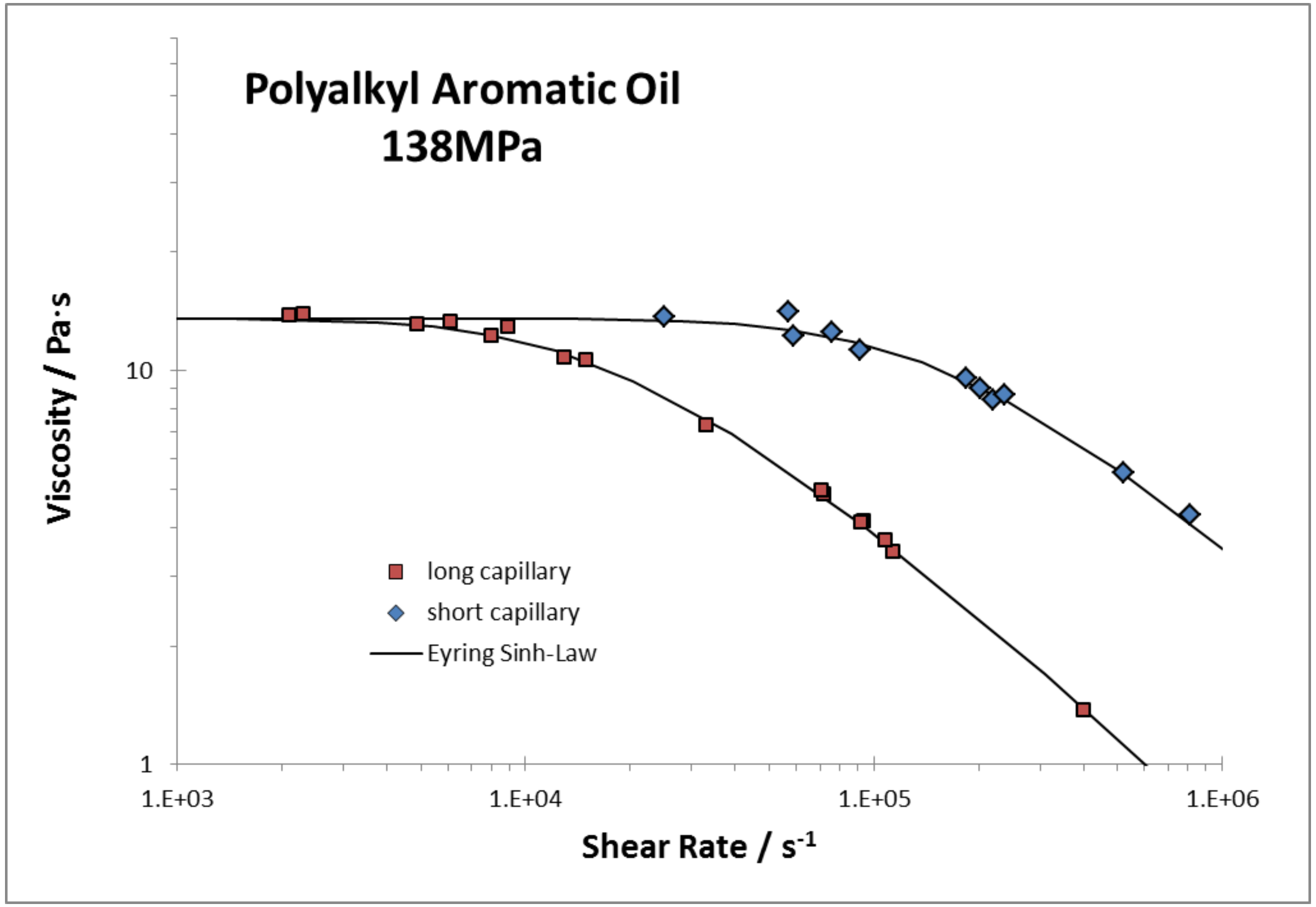
- **High Shear Stress** → **Shear Thinning** ✓
- **High Shear Rate** → **Thermal Softening** ✗
- **High Shear Stress at Ambient Pressure** → **Shear Cavitation** ✗

- **Elevated pressure enables high shear stress without high shear rate due to the large increase in viscosity.**
- **Elevated pressure removes the possibility of a tensile principal stress, the cause of shear cavitation.**

Constitutive behavior excludes the effect on viscosity of heat generated by the flow. This thermal softening is not always easy to recognize.

The inability to recognize the Eyring sinh-law as a thermal effect has had a disastrous effect on classical EHL, delaying the understanding of film-forming and friction for decades.

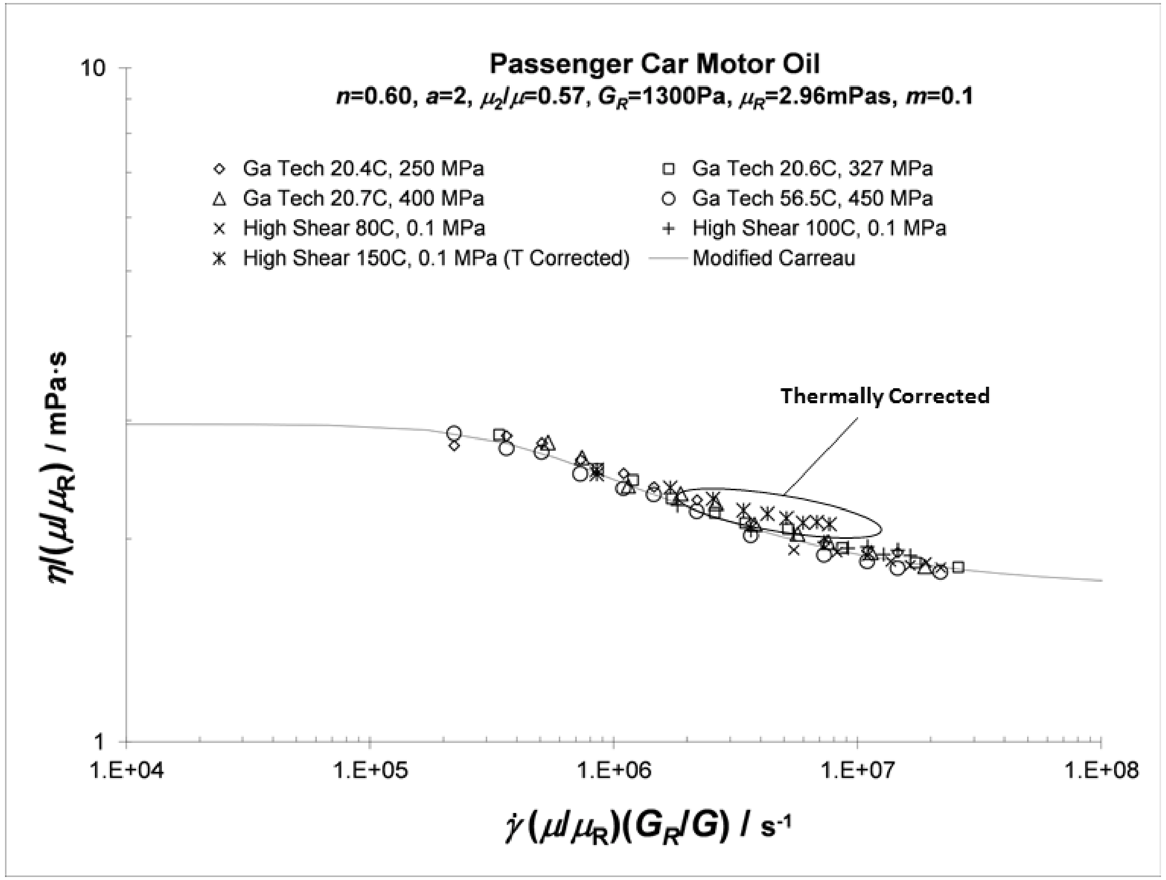
Here, the viscosity of a synthetic base oil was measured in two viscometers yielding two very different flow curves. The apparent shear-thinning is precisely represented by the sinh-law with different values of "Eyring stress."



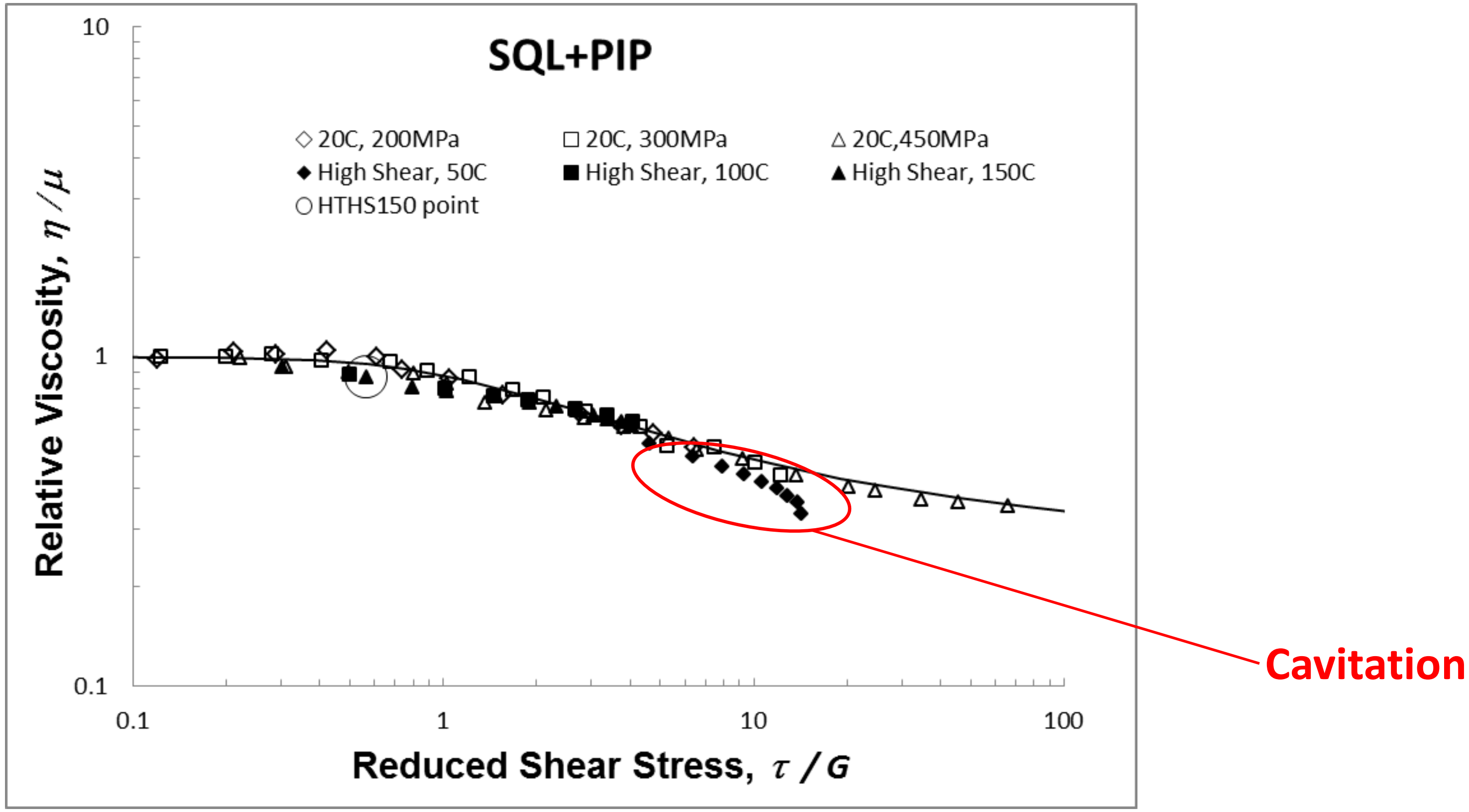
If it were possible to thermally correct shear-thinning viscosity for the temperature rise in an ambient pressure viscometer, it would be possible to extend flow curves to greater shear rates.

However, any calculation of the heat generated would require knowledge of the constitutive behavior. If that were known there would be no reason to do the measurement.

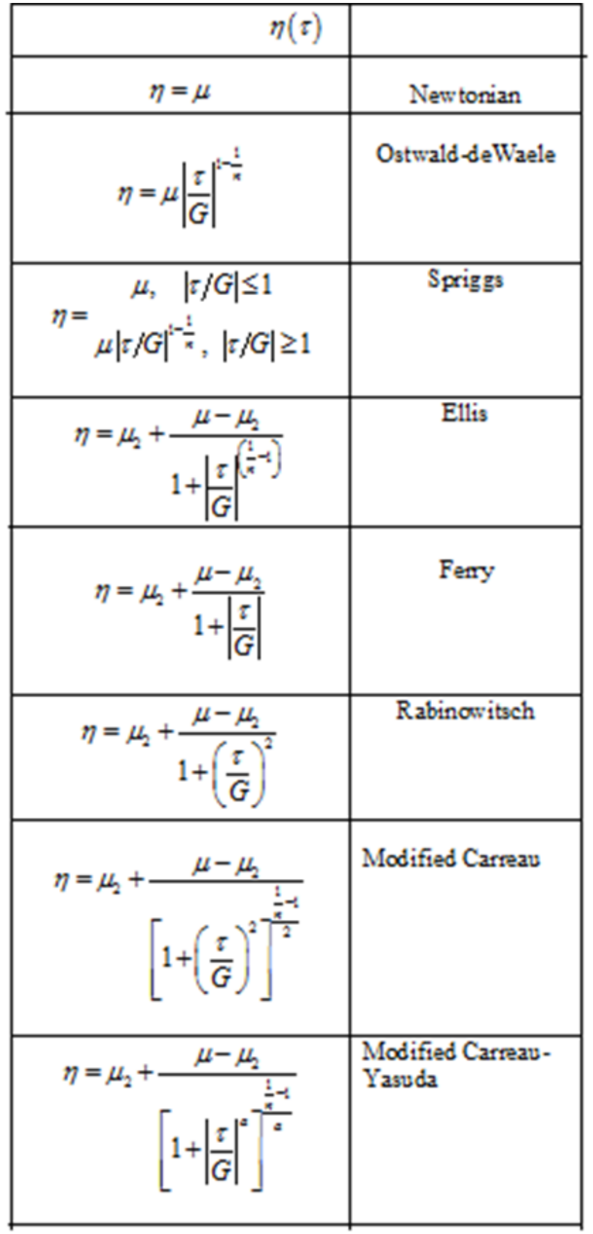
The thermally corrected data suggest a second Newtonian viscosity that is too large.



Flowing liquids do not readily support a tensile stress without rupture. Non-Newtonian flow results in extra tensile stress in the flow direction (first normal stress difference) so that shearing under ambient pressure will cavitate for shear stress less than 10^5 Pa=1 atmosphere.



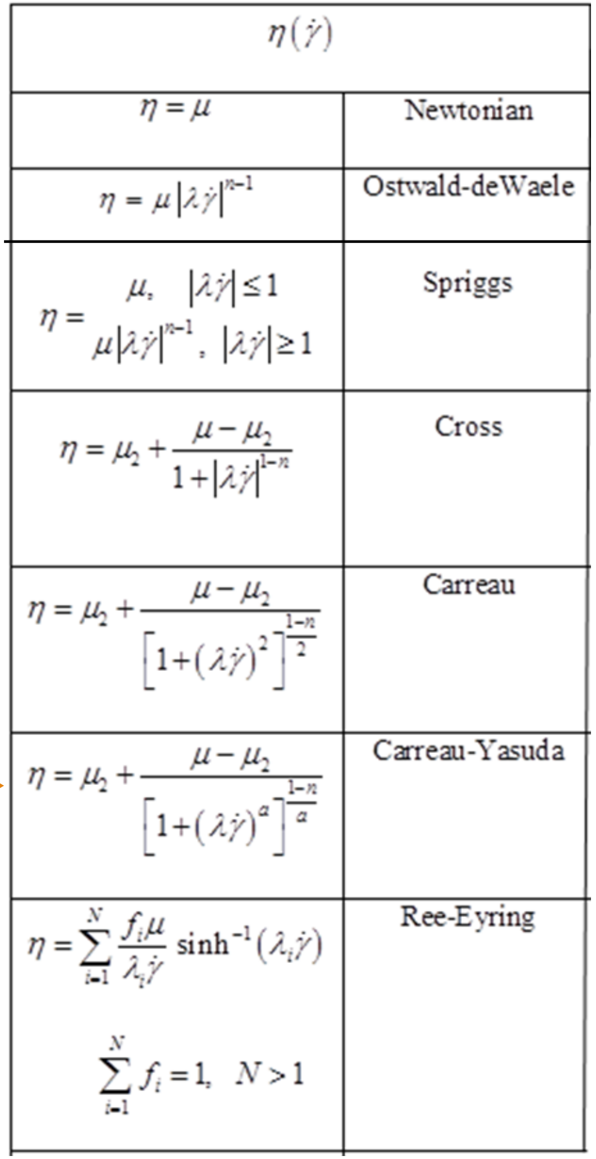
There are many viscosity functions.



$\eta(\tau)$	
$\eta = \mu$	Newtonian
$\eta = \mu \left\lvert \frac{\tau}{G} \right\rvert^{1-\frac{1}{n}}$	Ostwald-deWaele
$\eta = \begin{matrix} \mu, & \lvert \tau/G \rvert \le 1 \\ \mu \lvert \tau/G \rvert^{1-\frac{1}{n}}, & \lvert \tau/G \rvert \ge 1 \end{matrix}$	Spriggs
$\eta = \mu_2 + \frac{\mu - \mu_2}{1 + \left\lvert \frac{\tau}{G} \right\rvert^{\left(\frac{1}{n}-1\right)}}$	Ellis
$\eta = \mu_2 + \frac{\mu - \mu_2}{1 + \left\lvert \frac{\tau}{G} \right\rvert}$	Ferry
$\eta = \mu_2 + \frac{\mu - \mu_2}{1 + \left(\frac{\tau}{G} \right)^2}$	Rabinowitsch
$\eta = \mu_2 + \frac{\mu - \mu_2}{\left[1 + \left(\frac{\tau}{G} \right)^2 \right]^{\frac{\frac{1}{n}-1}{2}}}$	Modified Carreau
$\eta = \mu_2 + \frac{\mu - \mu_2}{\left[1 + \left\lvert \frac{\tau}{G} \right\rvert^a \right]^{\frac{\frac{1}{n}-1}{a}}}$	Modified Carreau-Yasuda

Viscosity Functions with stress as the independent variable

Viscosity Functions with rate as the independent variable



$\eta(\dot{\gamma})$	
$\eta = \mu$	Newtonian
$\eta = \mu \lvert \lambda \dot{\gamma} \rvert^{n-1}$	Ostwald-deWaele
$\eta = \begin{matrix} \mu, & \lvert \lambda \dot{\gamma} \rvert \le 1 \\ \mu \lvert \lambda \dot{\gamma} \rvert^{n-1}, & \lvert \lambda \dot{\gamma} \rvert \ge 1 \end{matrix}$	Spriggs
$\eta = \mu_2 + \frac{\mu - \mu_2}{1 + \lvert \lambda \dot{\gamma} \rvert^{1-n}}$	Cross
$\eta = \mu_2 + \frac{\mu - \mu_2}{\left[1 + (\lambda \dot{\gamma})^2 \right]^{\frac{1-n}{2}}}$	Carreau
$\eta = \mu_2 + \frac{\mu - \mu_2}{\left[1 + (\lambda \dot{\gamma})^a \right]^{\frac{1-n}{a}}}$	Carreau-Yasuda
$\eta = \sum_{i=1}^{N} \frac{f_i \mu}{\lambda_i \dot{\gamma}} \sinh^{-1}(\lambda_i \dot{\gamma})$ $\sum_{i=1}^{N} f_i = 1, \quad N > 1$	Ree-Eyring

The curves drawn through the valid data in the previous slides are viscosity functions of the form $\eta(\dot{\gamma})$ or $\eta(\tau)$.

It is extremely helpful to define the parameters in a consistent manner.

Viscosity is $\eta = \tau/\dot{\gamma}$.

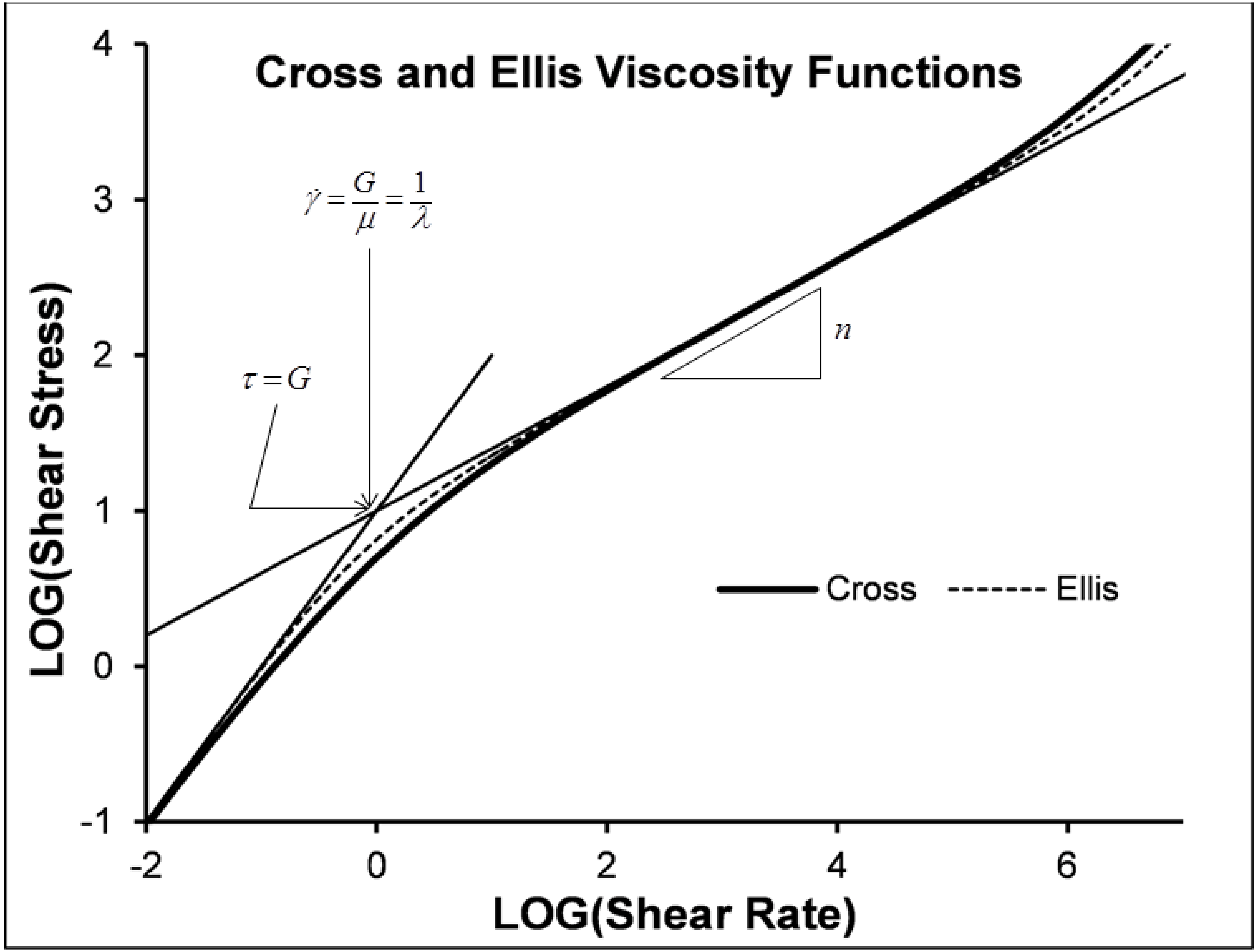
The limiting low-shear viscosity, μ , is equal to η for τ =0 or $\dot{\gamma}$ =0.

The power-law exponent is $n = d \ln \tau / d \ln \dot{\gamma}$ at infinite shear rate assuming $\mu_2 = 0$.

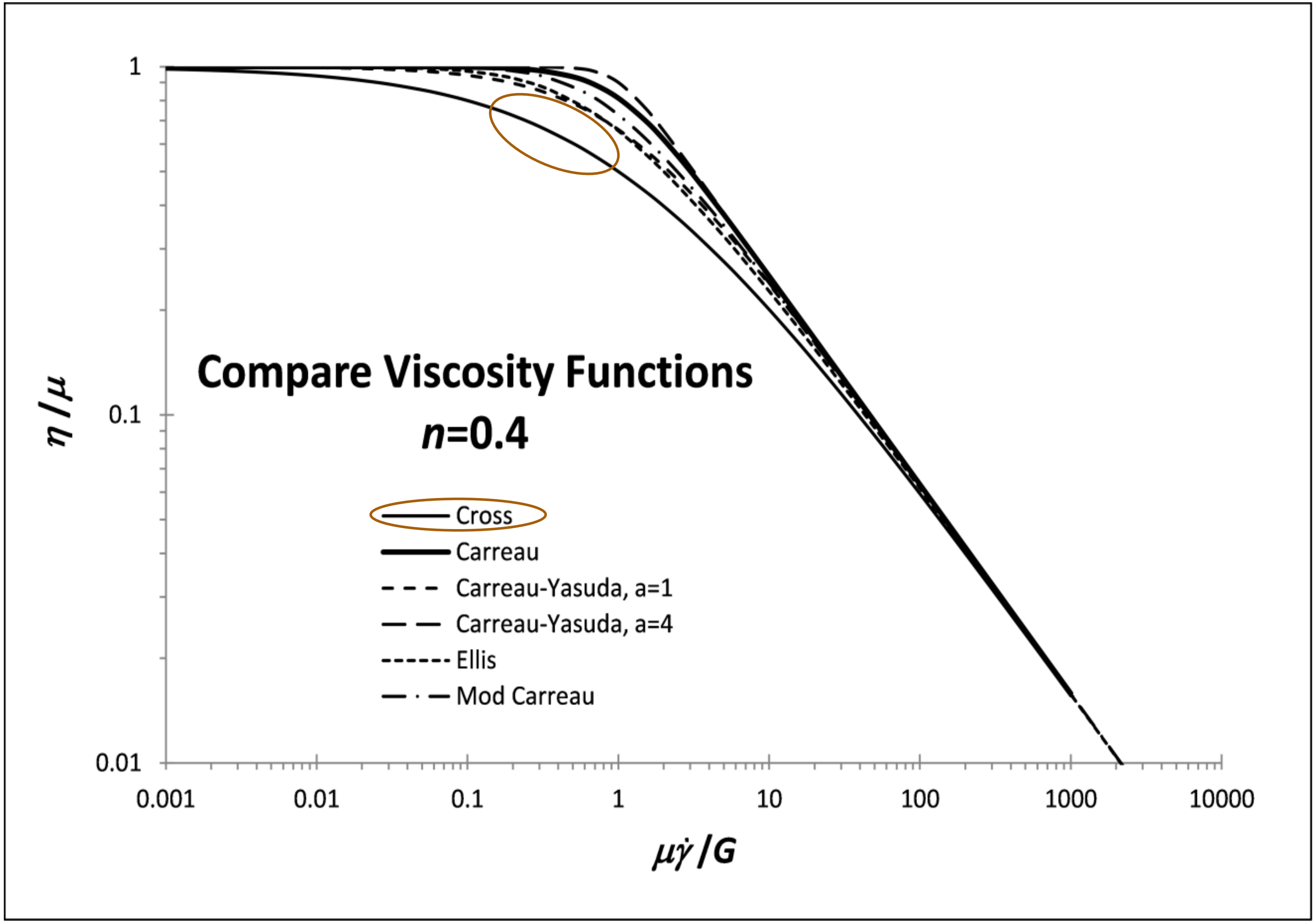
The characteristic time, λ , is the reciprocal of shear rate at the intersection of the extrapolated power-law and terminal regimes.

$G = \mu/\lambda$ is the stress at the intersection of extrapolated power-law and terminal regimes.

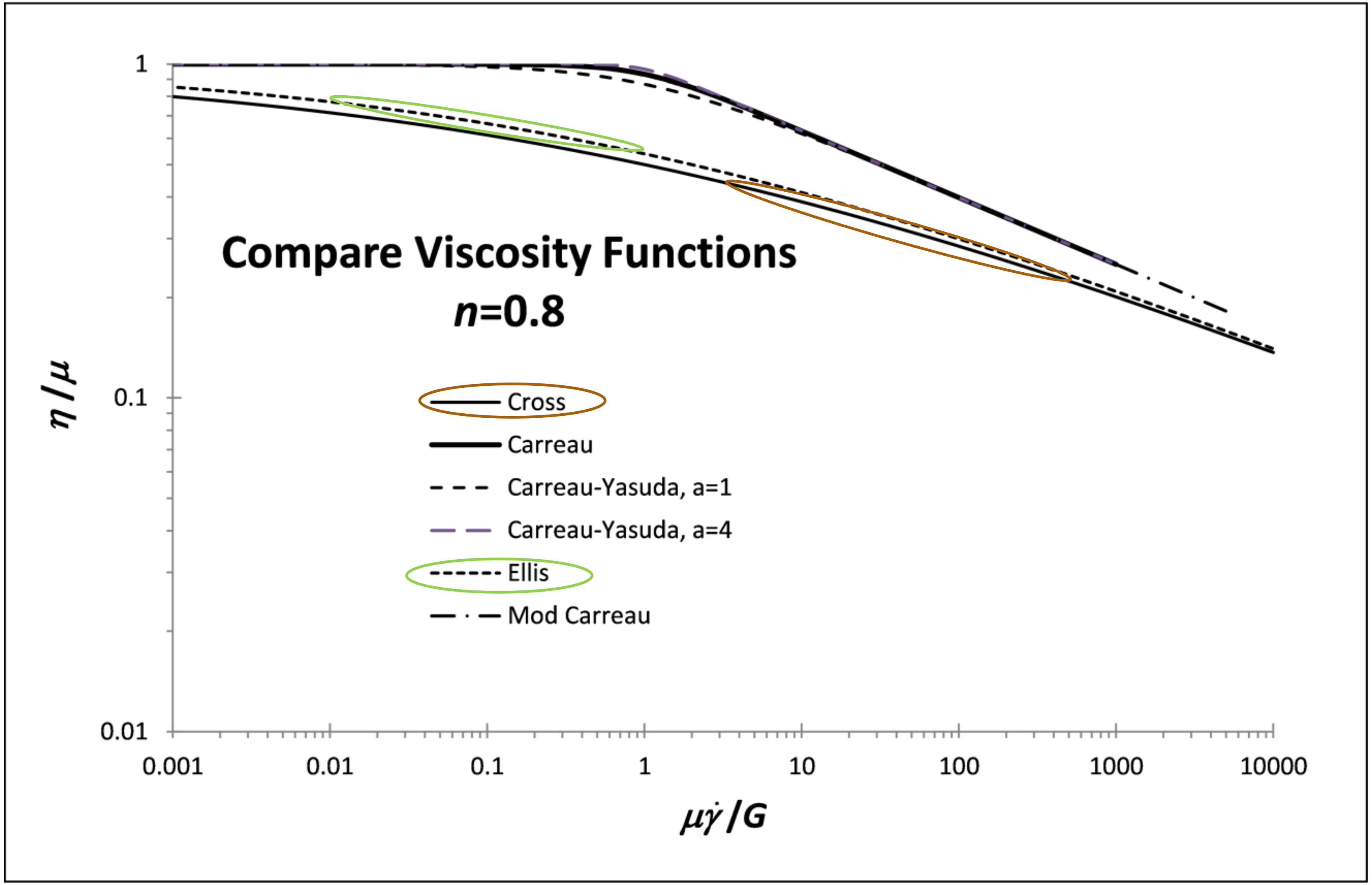
It is extremely helpful to define the parameters in a consistent manner.



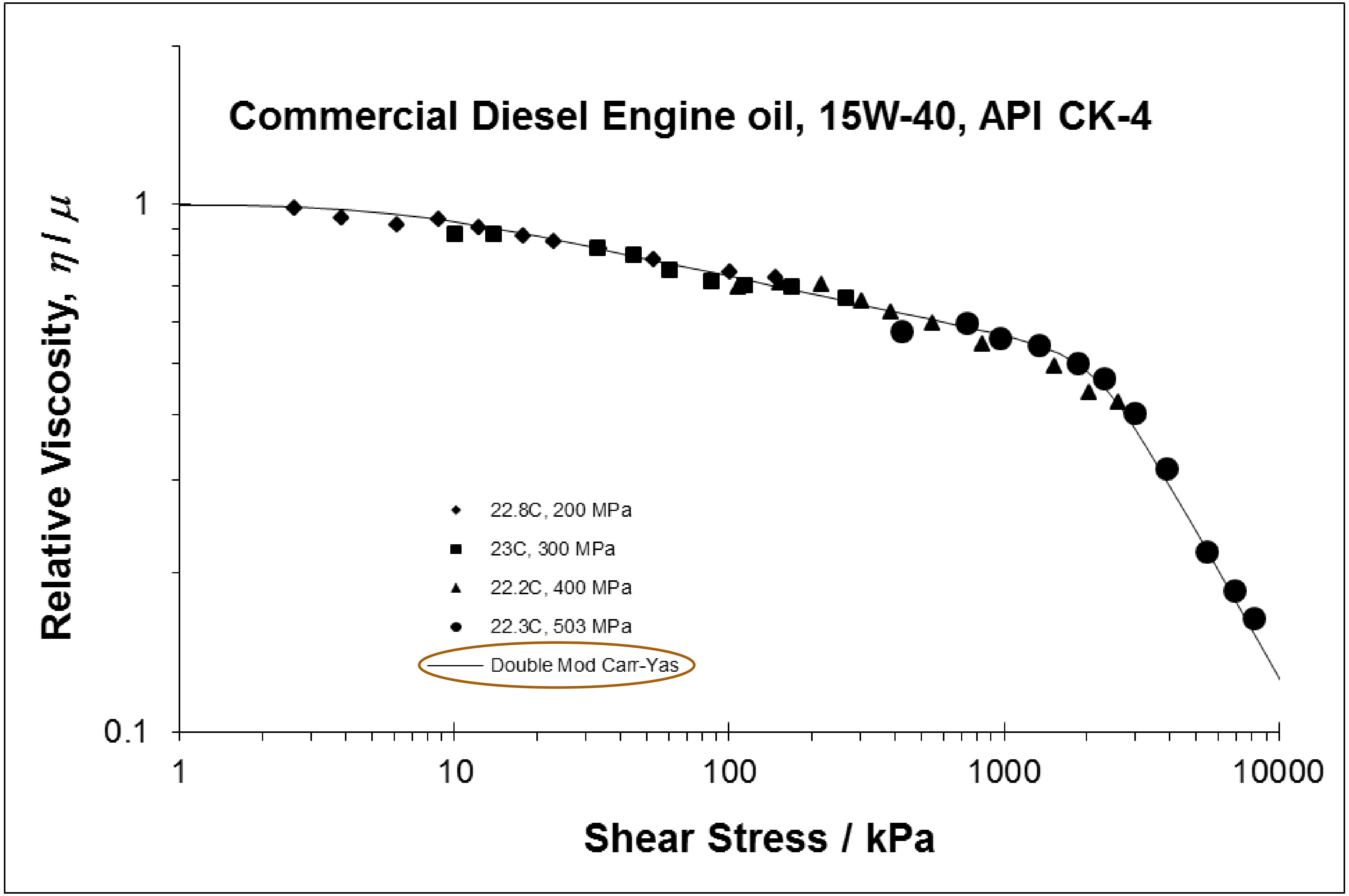
Cross has a very broad transition ≈ 3 decades



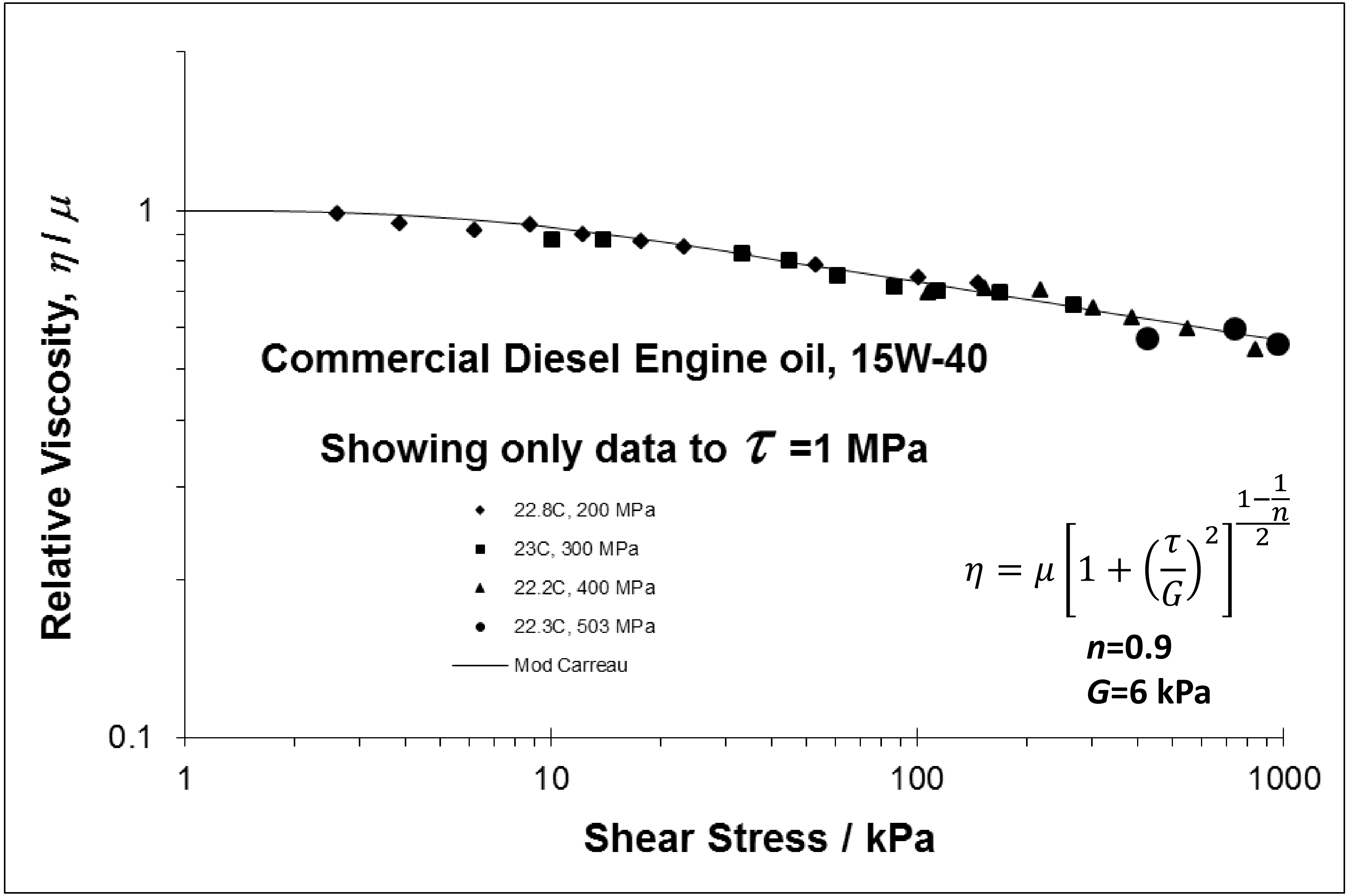
Cross and Ellis have extremely broad transitions, > 7 decades



Polymer-thickened oils do not have such broad transitions.


Commercial Diesel Engine oil, 15W-40, API CK-4
Relative Viscosity, η / μ
1
0.1
1
10
100
1000
10000
Shear Stress / kPa
◆ 22.8C, 200 MPa
■ 23C, 300 MPa
▲ 22.2C, 400 MPa
● 22.3C, 503 MPa
Double Mod Carr-Yas

The modified Carreau fits to τ = 1 MPa.

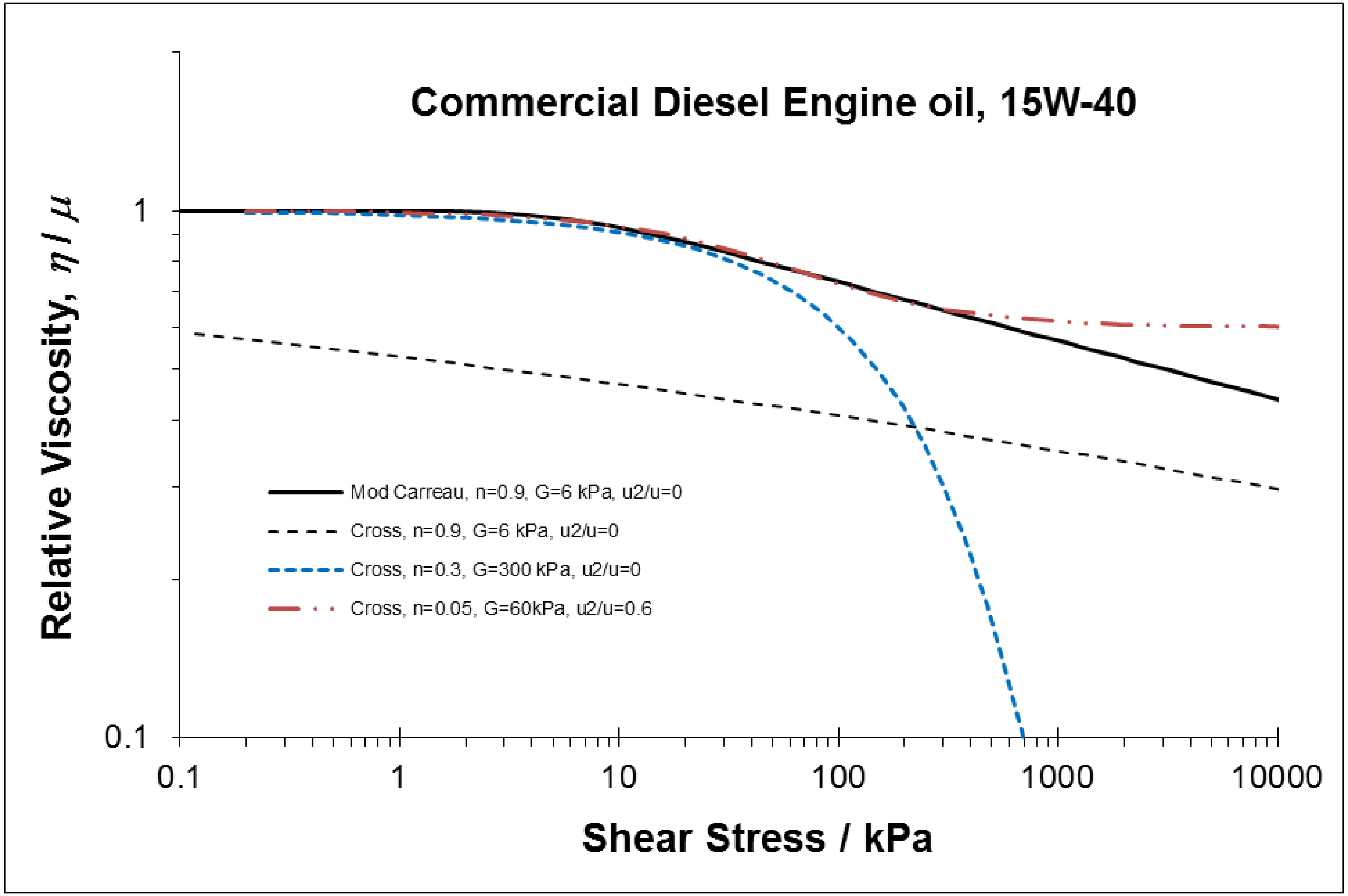


$$\eta = \mu\left[1+\left(\frac{\tau}{G}\right)^2\right]^{\frac{1-\frac{1}{n}}{2}}$$

n=0.9

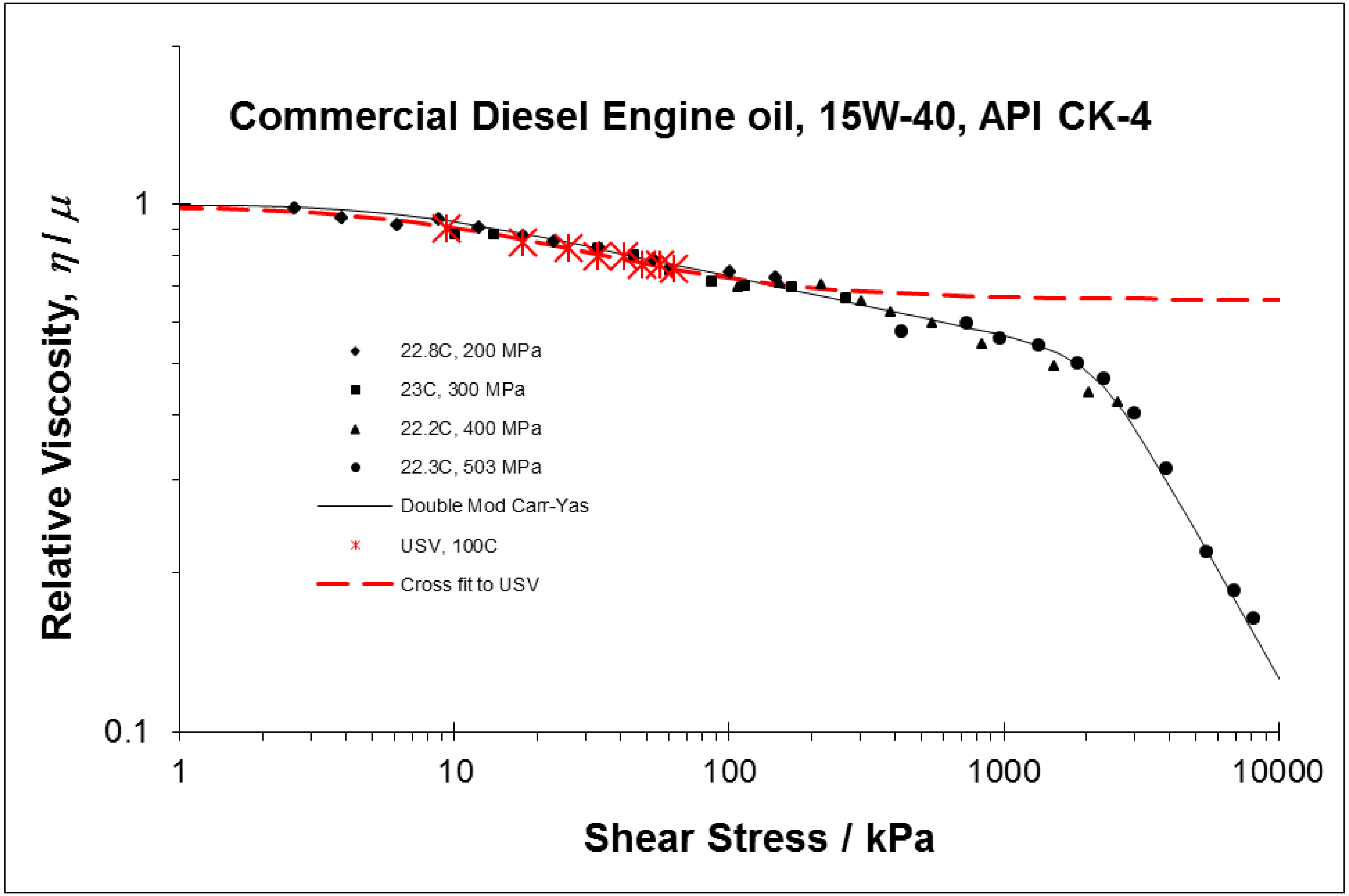
G=6 kPa

Attempt to fit the Cross equation.



Requires small *n*, large *G* and a second Newtonian.

To fit Cross to the ambient pressure, high shear data requires small *n*, large *G* and a second Newtonian which does not exist.

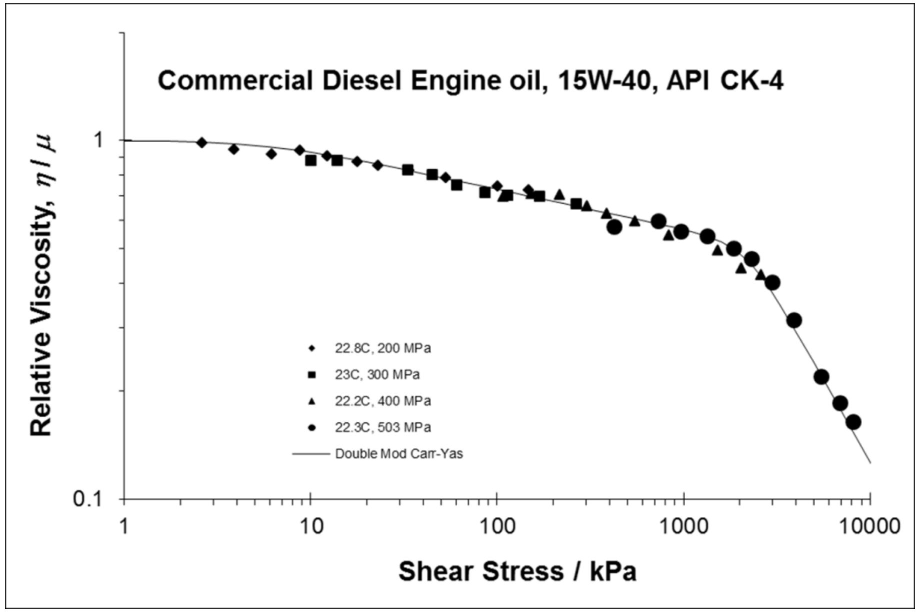


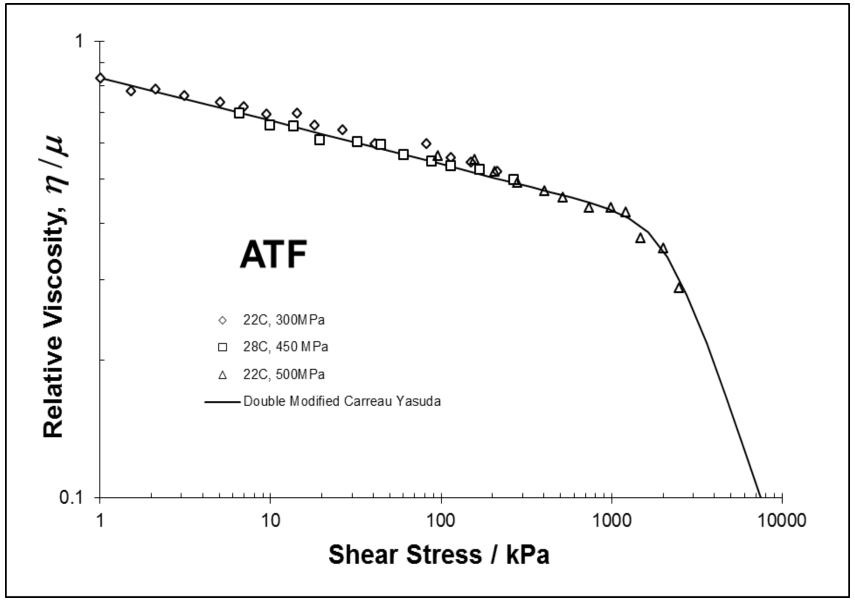
The Double Modified Carreau-Yasuda model does fit all of the available data without the unrealistic artifacts of other models.

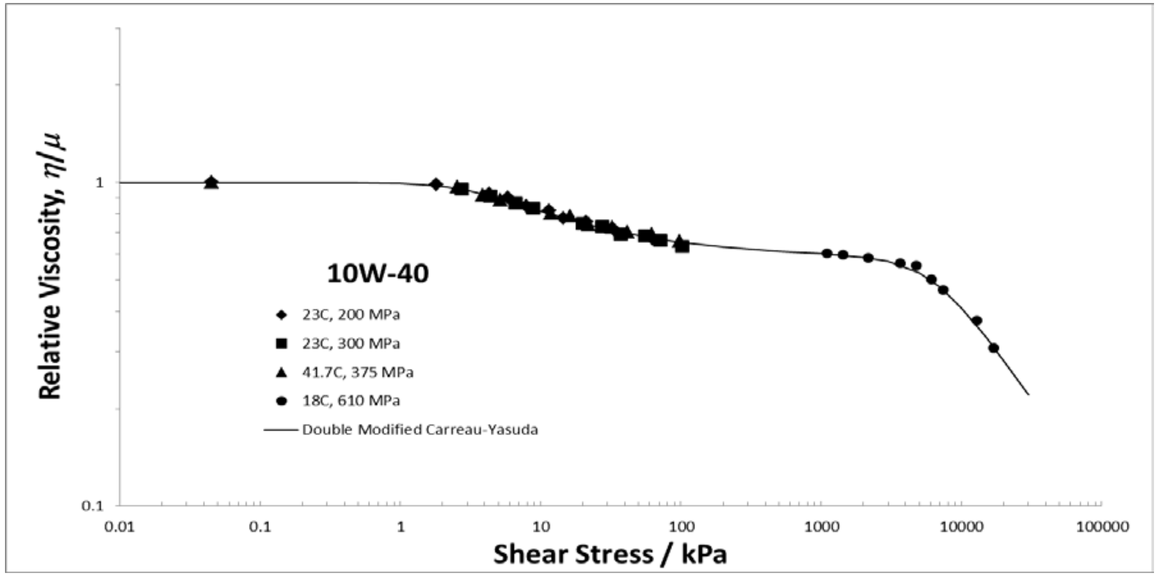
$$\frac{\eta}{\mu}=\left\{\frac{\mu_2}{\mu}+\left(1-\frac{\mu_2}{\mu}\right)\left[1+\left(\frac{\tau}{G_1}\right)^{a_1}\right]^{\frac{\left(1-\frac{1}{n_1}\right)}{a_1}}\right\}\left[1+\left(\frac{\tau}{G_2}\right)^{a_2}\right]^{\frac{\left(1-\frac{1}{n_2}\right)}{a_2}}$$

Designation	i	a_i	n_i	G_i /Pa	μ_2/μ
10W-40	1	2	0.66	3.44×10^3	0.58
	2	2.5	0.62	3.2×10^6	
ATF	1	2	0.915	1.5×10^2	0
	2	4	0.47	2.0×10^6	
15W-40	1	2	0.90	6.0×10^3	0
	2	5	0.55	2.2×10^6	

The Double Modified Carreau-Yasuda model fits all of the available data with parameters from the table.







CONCLUSIONS

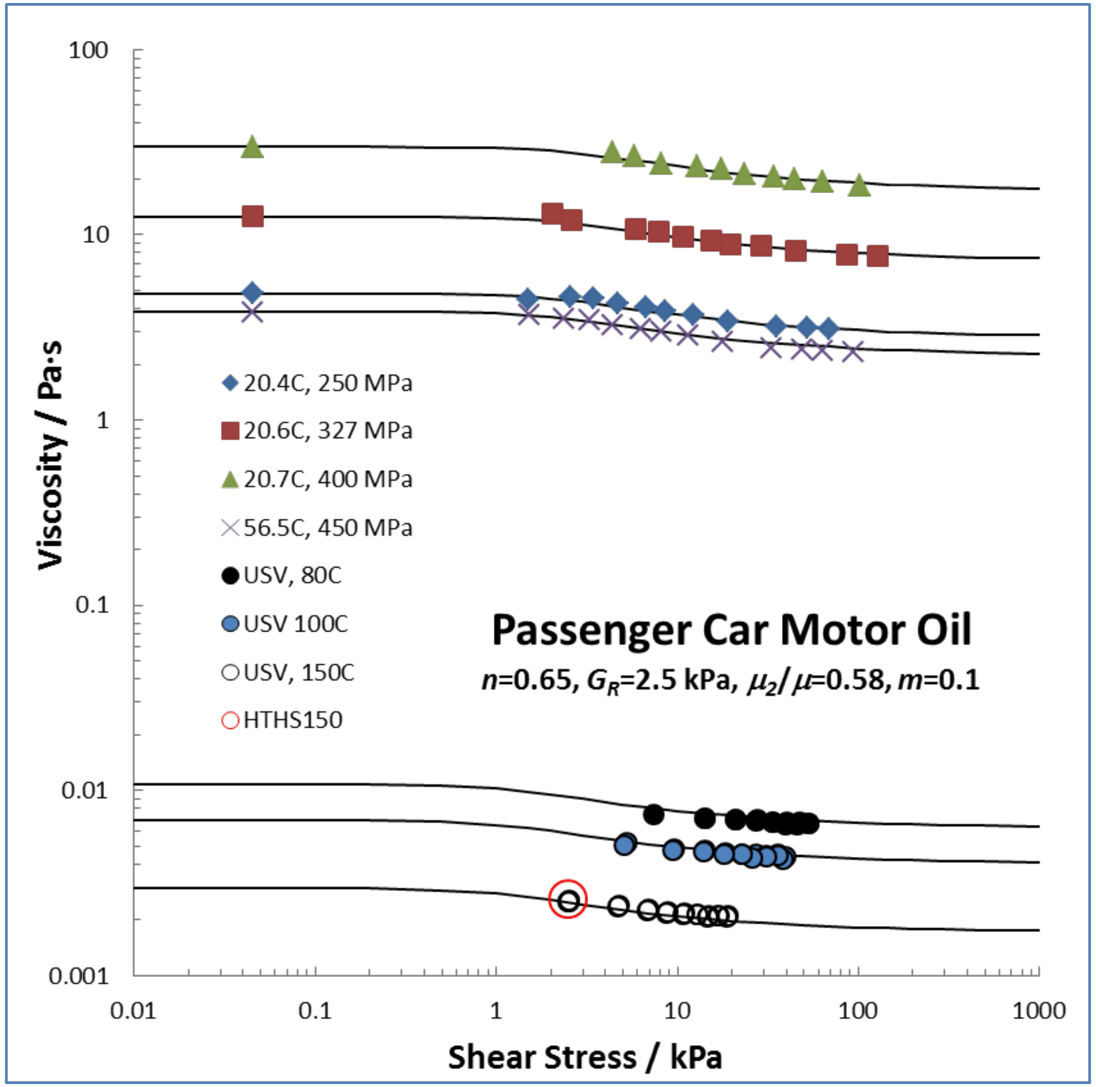
It is not possible to fully characterize the shear-thinning of a polymer-thickened oil in an ambient pressure viscometer. The HTHS 150 viscosity only provides information near the start of the transition to shear dependence.

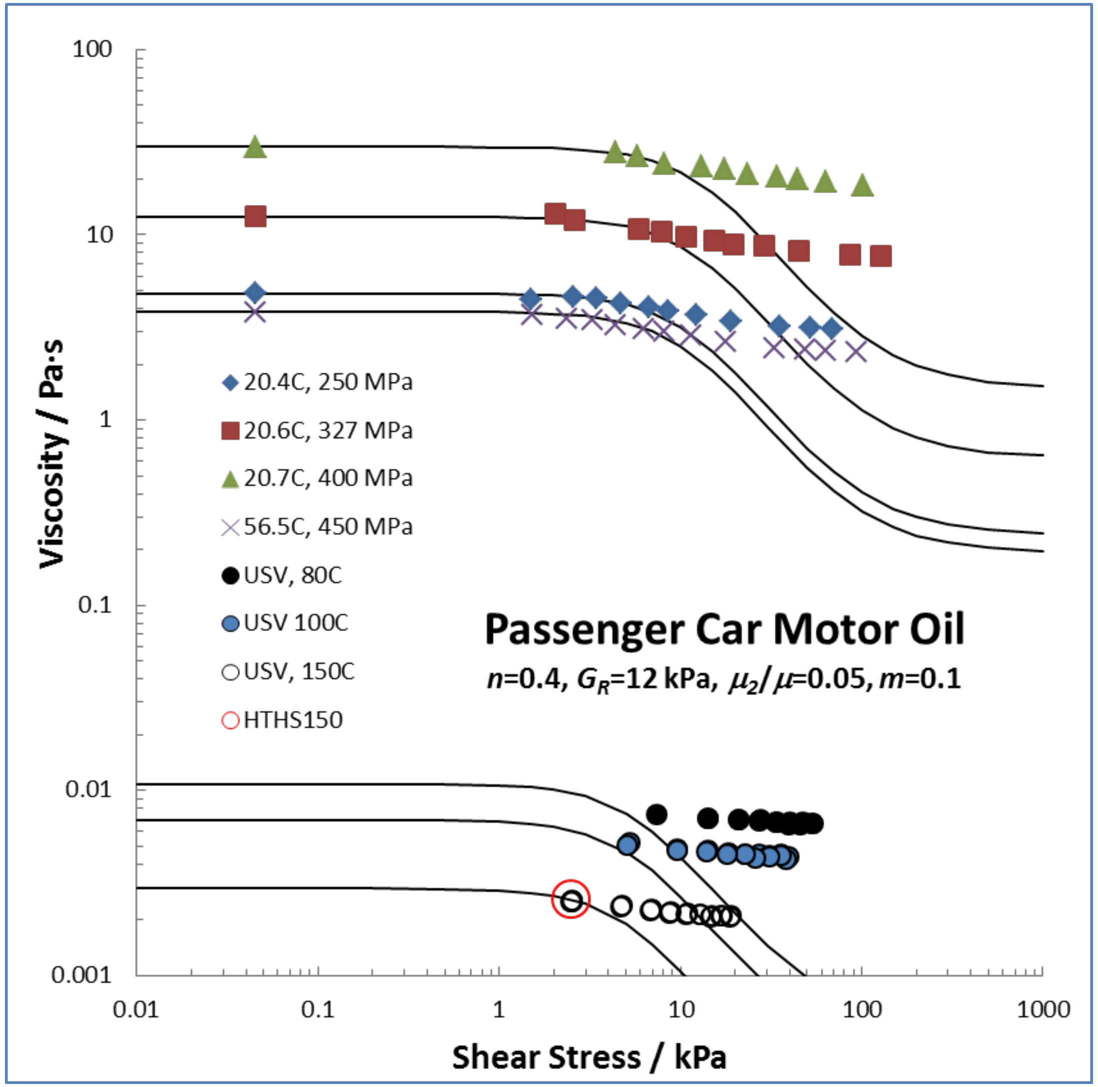
There is an expectation within the lubrication field that there must be a second Newtonian when a polymer is present. The application of the Cross model to ambient pressure data supports this notion even when a second Newtonian is not present and any success must therefore be fortuitous.

Multiplying separate Carreau-Yasuda terms for the polymer and for the base oil accurately describes the shear dependence for all of the data without the undesirable artifacts of other models.

CONCLUSIONS continued

There is also an expectation within the lubrication field that the second Newtonian viscosity is equal to the base oil viscosity. This is seldom an accurate assumption. The second Newtonian viscosity, if it exists, is usually greater than that of the base oil and rarely it is less.


Passenger Car Motor Oil
n=0.65, G_R=2.5 kPa, μ_2/μ=0.58, m=0.1
Viscosity / Pa·s
Shear Stress / kPa
100
10
1
0.1
0.01
0.001
0.01
0.1
1
10
100
1000
◆ 20.4C, 250 MPa
■ 20.6C, 327 MPa
▲ 20.7C, 400 MPa
× 56.5C, 450 MPa
● USV, 80C
● USV 100C
○ USV, 150C
○ HTHS150


Passenger Car Motor Oil
n=0.4, G_R=12 kPa, μ_2/μ=0.05, m=0.1
Viscosity / Pa·s
Shear Stress / kPa
100
10
1
0.1
0.01
0.001
0.01
0.1
1
10
100
1000
◆ 20.4C, 250 MPa
■ 20.6C, 327 MPa
▲ 20.7C, 400 MPa
× 56.5C, 450 MPa
● USV, 80C
● USV 100C
○ USV, 150C
○ HTHS150